The strain-modulated structure concurrent with the nucleation of the stripe domains

1989 J. Phys.: Condens. Matter 1 5759

(http://iopscience.iop.org/0953-8984/1/33/020)

J. Phys.: Condens. Matter **1** (1989) 5759–5765. Printed in the UK

The strain-modulated structure concurrent with the nucleation of the stripe domains

M Gajdek†, W Wasilewski‡ and G A Gehring§

† Department of Physics, Technical University, Aleja Tysiaclecia, Państwa Polskiego 7, 25-314 Kielce, Poland

‡ Department of Physics, Technical University, Malczewskiego 29, 26-600 Radom, Poland

§ Department of Theoretical Physics, University of Oxford, 1 Keble Road, Oxford OX1 3NP, UK

Received 19 December 1988

Abstract. The static properties of a thin ferromagnetic film near the phase transition point from the state of homogeneous magnetisation and deformation to the state of domain and modulated structure are considered. The phase transition is caused by a change in the external in-plane magnetic field. The strain-modulated structure concurrent with the nucleation of the stripe domains is described by means of the simplified perturbation scheme for solving the magneto-elastic equilibrium equations.

We consider a ferromagnetic sample in the shape of a thin film. The sample is a uniaxial ferromagnet made in such a way that the easy-magnetisation axis is perpendicular to the film plane. Let us assume that the in-plane sample dimensions are large enough that they can be treated as infinite in comparison with the film thickness L. The Cartesian coordinate system has been chosen in such a manner that the z axis is perpendicular to the film plane; it coincides with the direction of the easy-magnetisation axis. The energy constant of the uniaxial volume anisotropy of the ferromagnet is less than the maximum value of the demagnetising energy, i.e. $K < 2\pi M_s^2$. The considerations are confined to the low-temperature region $T \ll T_c$ where the magnitude $M_s = |M|$ of the magnetisation vector may be assumed to be constant.

In the presence of an external in-plane magnetic field applied in the y axis direction and of value greater than critical value $H_c(L)$, the sample is homogeneously magnetised in the field direction, i.e. $\boldsymbol{M} = \boldsymbol{M}^0 = [0, M_s, 0]$. We shall call this magnetisation state the ground state. In real ferromagnetic materials, which are also elastic, the ground state of magnetisation is accompanied by a ground state of deformation. The latter appears, even without external forces, as a result of the magneto-elastic coupling in ferromagnets [1–3].

For films of thickness greater than the critical value L_c, i.e. for $L > L_c(H = 0)$, the decrease in the external magnetic field below the critical value leads, at $H = H_c(L)$, to the occurrence of the magnetic phase transition. It is the transition from the above-mentioned ground state of homogeneous magnetisation to the state with the stripe domain structure [3–8]. An analysis of the energy of a thin ferromagnetic film, including magneto-elastic interactions with hexagonal symmetry, has been performed [9, 10]. The static and dynamic characteristics of the system given in [9, 10] indicate that the magnetic phase transition is accompanied by a structural phase transition from the ground state of homogeneous deformation to some kind of modulated structure.

Our aim in this paper is to attempt to describe the strain-modulated structure concurrent with the nucleation of domains. It is a very complicated problem [2, 3] because of the non-linearity of the coupled magnetic and mechanical equilibrium equations which ought to be solved with respect to the magnetisation vector $\boldsymbol{M}$ and displacement vector $\boldsymbol{u}$. For this reason, we confine our investigation of the problem to the vicinity of the phase transition point, i.e. for the magnetic field intensities obeying the condition $H_c - H \ll H_c$. Under the above restriction for the external magnetic field the magnetisation $\boldsymbol{M}$ and the deformation $e_{ij} \equiv \frac{1}{2}(u_{i,j} + u_{j,i})$ can be described by their small deviations from the ground state:

$$\boldsymbol{\mu} = \boldsymbol{m} - \boldsymbol{m}^0 \simeq [\mu_x, -\tfrac{1}{2}(\mu_x^2 + \mu_z^2), \mu_z] \qquad \varepsilon_{ij} = e_{ij} - e_{ij}^0 \tag{1}$$

where $\boldsymbol{m} = M_s^{-1}\boldsymbol{M}$ and $i, j = x, y, z$. Use of the small values $|\mu_i| \ll 1$ and $|\varepsilon_{ij}| \ll 1$ will enable us to make some simplifications in the equilibrium equations.

To obtain the equations for $\boldsymbol{\mu}$ and $\boldsymbol{u}$, we employ the so-called conventional magnetostriction theory used by many researchers and also in our previous work [9, 10]. Within this phenomenological theory the energy of the ferromagnet is presented as a sum of terms describing the elastic, magneto-elastic and magnetic energies:

$$F\{m_i, m_{i,j}, e_{ij}\} = F^{\mathrm{e}}\{e_{ij}\} + F^{\mathrm{me}}\{m_i, e_{ij}\} + F^{\mathrm{m}}\{m_i, m_{i,j}\}. \tag{2}$$

The first two terms in (2) for the uniaxial ferromagnet with a tetragonal or hexagonal lattice are usually written in the following form:

$$F^{\mathrm{e}}\{e_{ij}\} = \int_V \mathrm{d}\boldsymbol{r}\,[\tfrac{1}{2}C_{11}(e_{xx}^2 + e_{yy}^2) + C_{12}e_{xx}e_{yy} + \tfrac{1}{2}C_{33}e_{zz}^2 + C_{13}(e_{xx} + e_{yy})e_{zz} + 2C_{66}e_{xy}^2 + 2C_{44}(e_{xz}^2 + e_{yz}^2)] \tag{3a}$$

$$F^{\mathrm{me}}\{m_i, e_{ij}\} = \int_V \mathrm{d}\boldsymbol{r}\,[B_1(e_{xx} + e_{yy}) + B_3e_{zz} + (B_{11}m_x^2 + B_{12}m_y^2)e_{xx} + (B_{12}m_x^2 + B_{11}m_y^2)e_{yy} + B_{33}m_z^2e_{zz} + 2B_{66}m_xm_ye_{xy} + 2B_{44}m_z(m_xe_{xz} + m_ye_{yz})] \tag{3b}$$

where the abbreviated Voigt notation has been used for the fourth-rank tensors C_{ijkl} and B_{ijkl}—the elastic and magneto-elastic material constants, respectively. For the hexagonal crystal lattice, we assume further that the following equalities are fulfilled:

$$2C_{66} = C_{11} - C_{12} \qquad B_{66} = B_{11} - B_{12}. \tag{4}$$

The magnetic energy of the uniaxial ferromagnet may be described by the functional

$$F^{\mathrm{m}}\{m_i, m_{i,j}\} = \int_V \mathrm{d}\boldsymbol{r}\,(Am_{i,j}m_{i,j} - Km_z^2 + K_2m_z^4 - \tfrac{1}{2}M_s^2h_i^dm_i - M_s^2h_im_i) + \int_S K_sm_z^2\,\mathrm{d}S \tag{5}$$

where $h_i = M_s^{-1}H_i^{\mathrm{ext}} = [0, h, 0]$ is the reduced magnetic field, $h_i^{\mathrm{d}} = M_s^{-1}H_i^{\mathrm{d}}$ is the demagnetisation field and $m_{i,j} \equiv \partial m_i/\partial x_j$. A is the exchange energy constant. K and K_2 are the volume anisotropy constants and K_s is the surface anisotropy constant. Magnetostatic Maxwell's equations connect the demagnetisation field $\boldsymbol{h}^d$ and magnetisation $\boldsymbol{m}$:

$$\varepsilon_{ijk} h^d_{j,k} = 0 \qquad (h^d_i + 4\pi m_i)_i = 0 \tag{6}$$

where the convention of summing over repeated indexes is applied. The ground state of deformation e^0_{ij} for assumed homogeneous magnetisation $\boldsymbol{m}^0 = [0, 1, 0]$ is determined by the equation

$$(\delta F/\delta e_{ij})|_{e^0_{ij}, m^0_i} = 0. \tag{7}$$

The explicit form of the formulae for the e^0_{ij} tensor components can be found in [9] or [11].

Evaluating the total free energy F, given by (2), (3) and (5), in the vicinity of the ground state $\boldsymbol{m}^0$, e^0_{ij} and using (1), we obtain (for the part connected with deviations $\boldsymbol{\mu}$ and ε_{ij}) the following approximative expression:

$$\begin{aligned}
\delta F &= \int_V f(\mu_i, \mu_{i,j}, \varepsilon_{ij})\, \mathrm{d}\boldsymbol{r} + \int_S f_s(\mu_i)\, \mathrm{d}S \\
&= \int_V \mathrm{d}\boldsymbol{r} \{ \tfrac{1}{2} C_{11}(\varepsilon^2_{xx} + \varepsilon^2_{yy}) + C_{12}\varepsilon_{xx}\varepsilon_{yy} + \tfrac{1}{2} C_{33}\varepsilon^2_{zz} \\
&\quad + C_{13}(\varepsilon_{xx} + \varepsilon_{yy})\varepsilon_{zz} + 2C_{66}\varepsilon^2_{xy} + 2C_{44}(\varepsilon^2_{yz} + \varepsilon^2_{xz}) \\
&\quad + 2B_{44}\mu_z\varepsilon_{yz} + 2B_{66}\mu_x\varepsilon_{xy} + (B^2_{44}/2C_{44})\mu^2_z + (B^2_{66}/2C_{66})\mu^2_x \\
&\quad - K^*\mu^2_z + K^*_2\mu^4_z + \tfrac{1}{2} M^2_s(\mu^2_x + \mu^2_z) - \tfrac{1}{2} M^2_s \boldsymbol{h}^d \boldsymbol{\mu} \\
&\quad + A[(\nabla\mu_x)^2 + (\nabla\mu_z)^2] \} + \int_S K_s \mu^2_z\, \mathrm{d}S
\end{aligned} \tag{8}$$

where K^* and K^*_2 are the volume anisotropy constants renormalised by magnetostriction. Further, we assume the two variable dependences $\boldsymbol{\mu} = \boldsymbol{\mu}(x, z)$ for the magnetisation and $\varepsilon_{ij} = \varepsilon_{ij}(x, z)$ for deformation, i.e. its spatial distributions are homogeneous in the y direction. From equilibrium equations of the form

$$\varepsilon_{ijk}\mu_j\, \delta F/\delta\mu_k = 0 \qquad (\delta F/\delta\varepsilon_{ij})_j = 0 \tag{9}$$

with the significant assumption of the term proportional to μ^3_z, we obtain the following set of differential equations:

$$\begin{aligned}
&\alpha\nabla^2\mu_z + (\beta^* - h)\mu_z + h^d_z - \tfrac{1}{2}(\beta^* + \beta^*_2)\mu^3_z \\
&\qquad - (B_{44}/C_{44}M^2_s)(C_{44}\, \partial u_y/\partial z + B_{44}\mu_z) = 0
\end{aligned} \tag{10}$$

$$\alpha\nabla^2\mu_x - h\mu_x + h^d_x - (B_{66}/C_{66}M^2_s)(C_{66}\, \partial u_y/\partial x + B_{66}\mu_x) = 0 \tag{11}$$

$$(\partial/\partial x)(C_{66}\, \partial u_y/\partial x + B_{66}\mu_x) + (\partial/\partial z)(C_{44}\, \partial u_y/\partial z + B_{44}\mu_z) = 0 \tag{12}$$

where the following definitions have been used:

$$\alpha \equiv 2A/M^2_s \qquad \beta^* \equiv 2K^*/M^2_s \qquad \beta^*_2 \equiv 8K^*_2/M^2_s. \tag{13}$$

The set of differential equations (10)–(12) should be solved simultaneously with the magnetostatic equations (6). We assume the solutions to fulfil the boundary conditions obtained by linearisation of the appropriate equilibrium equations on the surfaces of the thin film. When the surface anisotropy is of the easy-plane type (the hard axis is perpendicular to the film surfaces), the boundary conditions for the magnetisation vector have the form

$$(\partial\mu_z/\partial z \pm \mu_z/\Lambda_\pm)_{z=\pm L/2} = 0 \qquad (\partial\mu_x/\partial z)|_{z=\pm L/2} = 0 \tag{14}$$

where $\Lambda_\pm = \Lambda \equiv AK_s^{-1}$. The parameter Λ describes the degree of surface spin pinning; the spins are completely pinned when the surface anisotropy is extremely strong, $\Lambda = 0$. The mechanical boundary condition for the u_y component of the displacement vector when the surfaces are traction free takes the form

$$(C_{44}\,\partial u_y/\partial z + B_{44}\mu_z)_{z=\pm L/2} = 0. \tag{15}$$

In [8] the solutions $\mu_x(x, z)$, $\mu_z(x, z)$ of the set of equations (10) and (11) have been presented for zero magnetostriction. They are the same (with the exchange of K^* for K) as the solutions of equations (10) and (11) under the mechanical quasi-equilibrium condition $\partial F/\partial\varepsilon_{ij}$. The solution $\mu_z(x, z)$ has the following form:

$$\mu_z(x, z) = \mu_{0z}\,\mathrm{sn}(\bar{\kappa}x; p)\cos(kz) \underset{h\simeq h_c}{\longrightarrow} \mu_{0z}\sin(2\pi x/\lambda)\cos(kz) \tag{16}$$

whereas $\mu_x(x, z)$ is determined by the equation [8]

$$\partial\mu_x/\partial x = -[(4\pi - \beta^* + h)/(4\pi + h)]\,\partial\mu_z/\partial z \tag{17}$$

and has the form

$$\mu_x(x, z) = \mu_{0x}g(\bar{\kappa}x; p)\sin(kz) \underset{h\simeq h_c}{\longrightarrow} \mu_{0x}\cos(2\pi x/\lambda)\sin(kz). \tag{18}$$

The structure parameters μ_{0z}, μ_{0x}, λ, h_c and L_c are given in the Appendix. To satisfy the boundary conditions (14) by $\mu_x(x, z)$ and $\mu_z(x, z)$, the following equation for the parameter k must hold:

$$k\tan(\tfrac{1}{2}kL) = \Lambda^{-1}. \tag{19}$$

For relatively small values of $\Lambda \ll L_c$ there exists an approximate solution of (19) in the form

$$k \simeq \pi/(L + 2\Lambda). \tag{20}$$

The proposed method of estimation of the strains in the modulated structure consist of the performance of the first step in a perturbative scheme for the set of equations (10)–(12), i.e. the solution of equation (12) using the results given by (16) and (17) as the zero-order approximation. Inserting (17) and the approximate form of $\mu_z(x, z)$ given by (16) into (12) and (15), one can obtain the differential equation for the $u_y(x, z)$ component of the displacement vector and its boundary condition. The solution fulfilling the boundary condition may be described as the deviation of u_y from the ground-state value u_y^0 in the following form:

$$\Delta u_y(x, z) \equiv u_y(x, z) - u_y^0$$
$$= [\![-\vartheta_1\sinh[(2\pi\sqrt{C_{66}}/\lambda\sqrt{C_{44}})z] + \vartheta_0\sin\{[\pi/(L+2\Lambda)]^z\}]\!]\sin[(2\pi/\lambda)x]. \tag{21}$$

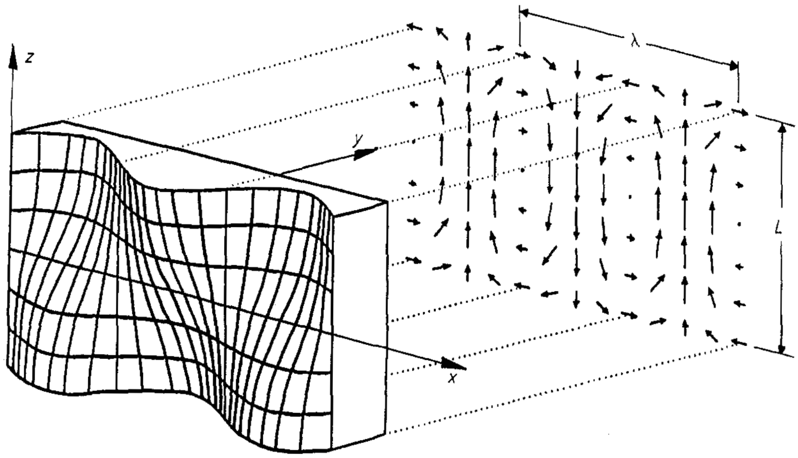

Figure 1. Schematic illustration of the displacements $\Delta u_y(x, z)$ in the modulated structure; in the second plane is shown the projection of the magnetisation vector on the (x, z) plane (perpendicular to the film surfaces) for stripe domain structure.

The amplitudes ϑ_0 and ϑ_1 are given by the following expressions:

$$\vartheta_0 = \{[(B_{66} - B_{44})(4\pi + h) - B_{44}\beta^*]/(C_{44}k^2 + C_{66}\kappa^2)(4\pi + h)]\}k\mu_{0z} \quad (22)$$

$$\vartheta_1 = (\vartheta_0 + B_{44}\mu_{0z}/C_{44}k)\pi\Lambda k/k_1 L \cosh(k_1 L/2) \quad (23)$$

where k is given by (20), $\kappa \equiv 2\pi/\lambda$ and $k_1 \equiv \kappa\sqrt{C_{66}}/\sqrt{C_{44}}$.

An illustration of the strain-modulated structure may be the surface shape which we obtain as a result of the displacements $\Delta u_y(x, z)$ (given by (21)) in the direction perpendicular to the cross section of the film. Figure 1 shows the surface for amplitude $\vartheta_0 > 0$, whereas in the second figure plane the stripe domain structure has been shown by plotting the projection of the magnetisation vector $\boldsymbol{m} = \boldsymbol{\mu} + \boldsymbol{m}^0$ on the (x, z) plane. The displacements $\Delta u_y(x, z)$ vanish at the critical point $h = h_c$ (where h_c is given by (A4)) together with the magnetisation components μ_x and μ_z because of the $\mu_{0z}, \mu_{0x} \sim (h_c - h)^{1/2}$ dependence near h_c (see (A1) and (A2)) and the $\vartheta_0, \vartheta_1 \sim \mu_{0z}$ proportionality. The deformation structure described by displacements $\Delta u_y(x, z)$ is periodical in the x direction with the period given by (A7) or in a more drastic approximation by the expression

$$\lambda(L, h) = \lambda_c(L)\{1 + [(4\pi - \beta^*)\lambda_c^4/16\beta^*(L_c + 2\Lambda)^2 L^2](h_c - h)/\beta^*\}. \quad (24)$$

The variation in the external magnetic field leads to variations in the periodicity and amplitude of the strains in the modulated structure.

It seems that one possibility for testing the existence of the strain-modulated structure is observation of the absorption effects for phonons which may interfere with this periodic structure. The other possibility is the effect of acoustic emission observed [12] in a ferromagnet in the domain nucleation phase during variation of the external magnetic field. Perhaps this effect is connected to the strain variation resulting from the field dependence of the period $\lambda(L, h)$ and amplitude $\vartheta_0(L, h)$ of the modulated structure associated with the magnetic domain structure.

Acknowledgment

Work has been supported in part by the University of Łódź, Poland, under Contract CPBP 01.08.

Appendix

If we consider more general magnetic boundary conditions and take into account the second-order anisotropy term, the domain structure parameters differ from these derived in [8]. We present here, for the parameters describing the stripe domain structure, the formulae including the surface spin pinning parameter $\Lambda = AK_s^{-1}$ and the second-order anisotropy constant K_2^*. The amplitudes μ_{0z}, μ_{0x} of the magnetisation components are scaled by the factor $s = K^*/(K^* + K_2^*)$; so they take the form

$$\mu_{0z}(L, h) = [\![s(\xi^2 - \xi_c^2)\{\xi - [(4\pi + \beta^*)/8\pi]\xi_c^2\}^{-1}]\!]^{1/2} \quad \text{(A1)}$$

$$\mu_{0x}(L, h) = [(4\pi - \beta^* + h)/2(4\pi + h)][s(4\pi + \beta^*)/8\pi]^{1/2} \times [(L_c + 2\Lambda)/(L + 2\Lambda)] \ln[(1 + p)/(1 - p)]. \quad \text{(A2)}$$

The amplitudes μ_{0z}, μ_{0x} tend to zero at $h = h_c$ proportionally to the factor $[(h_c - h)/\beta^*]^{1/2}$. In the above formulae, we use the reduced field parameter ξ and its critical value ξ_c:

$$\xi \equiv (\beta^* - h)/\beta^* \qquad \xi_c \equiv (\beta^* - h_c)/\beta^* \quad \text{(A3)}$$

where h_c is the critical value of the external magnetic field:

$$h_c(L) = \beta^* - \beta^*[1 - (\beta^*/4\pi)^2]^{-1}(L_c + 2\Lambda)/(L + 2\Lambda) \quad \text{(A4)}$$

and where L_c denotes the critical thickness of the film:

$$L_c = [\pi\alpha(4\pi - \beta^*)/\beta^*]^{1/2} - 2\Lambda. \quad \text{(A5)}$$

The symbol p that we use in (A2) denotes the elliptic integral modulus:

$$p^2 = (\xi^2 - \xi_c^2)\{\xi - [(4\pi + \beta^*)/8\pi]\xi_c^2\}^{-2}. \quad \text{(A6)}$$

The period of the domain structure which appears below the critical point $h = h_c$ may be expressed as follows:

$$\lambda(L, h) = 4K(p)\bar{\kappa}^{-1} = 4K(p)[\![(\beta^*/2\alpha)\{\xi - [(4\pi + \beta^*)/8\pi]\xi_c^2\}]\!]^{-1/2} \quad \text{(A7)}$$

where $K(p)$ is a complete elliptic integral of the first kind having the value $K(p) \simeq (\pi/2)(1 + \frac{1}{4}p^2 + \ldots)$ for small values of p ($\ll 1$). The critical value of the structure period λ depends on the film thickness:

$$\lambda_c(L) = 2d^{1/2}(L + 2\Lambda)/(L - d)^{1/2} \quad \text{(A8)}$$

where $d = \frac{1}{2}(4\pi + \beta^*)^{1/2}(4\pi - \beta^*)^{-1/2}(L_c + 2\Lambda)$.

References

[1] Brown W F 1963 *Micromagnetism* (New York: Wiley)
[2] Brown W F 1966 *Magnetoelastic Interactions* (Berlin: Springer)

[3] Hubert A 1974 *Theorie der Domänenwände in Geordnete Medien* (Berlin: Springer)
[4] Muller M W 1967 *J. Appl. Phys.* **38** 2413
[5] Tarasenko V V, Chensky E V and Dikstein I E 1976 *Zh. Eksp. Teor. Fiz.* **70** 2178
[6] Patashinsky A Z and Wasilewski W 1980 *Acta Phys. Pol.* A **57** 789
[7] Dikstein I E, Lisovsky F V, Mansvetova E G and Tarasenko V V 1984 *Zh. Eksp. Teor. Fiz.* **86** 1473
[8] Gajdek M and Wasilewski W 1983 *J. Magn. Magn. Mater.* **37** 246; 1985 *J. Phys. C: Solid State Phys.* **18** 3557
[9] Gehring G A, Wasilewski W and Gajdek M 1988 *J. Phys. C: Solid State Phys.* **21** 3545
[10] Wasilewski W, Gajdek M and Gehring G A 1989 *J. Physique Coll.* **49** C8 (Suppl. 12) 1951
[11] Vonsovsky C V and Turov E A (ed.) 1986 *Dynamic and Kinetic Properties of Magnetics* (Moscow: Nauka) (in Russian)
[12] Guyot M, Marceron T and Cagan V 1987 *Phys. Lett.* **84A** 64